7 Pages)

Reg. No. : ....

ode No. : 5333 **Sub. Code : PMAM 44**

I.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017 onwards)

me : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

A space having a countable dense subset is called a

(a) Lindelof space (b) Separable space

(c) Hausdorff space (d) Regular space

Which one of the following is not true

(a) A regular space is Hausdorff

(b) A normal space is regular

(c) A hausdorff space is regular

(d) A product of regular spaces is regular

3. Let X be a well-ordered set. Then every interval of the form $(x, y]$ is

(a) closed in X

(b) open in X

(c) neither open nor closed in X

(d) both open and closed in X

4. Consider the two statements

A : A normal space is completely regular

B : A completely regular space in regular. Then

(a) Both A and B are true

(b) Neither A nor B is true

(c) A is true but B is not true

(d) A is not true but B is true

5. Every ———— space X with a countable basis is metrizable.

(a) Hausdorff (b) Normal

(c) Regular (d) Topological

If we divide the internal $[-r, r]$ into three equal intervals of length $\frac{2}{3}r$, then the middle interval is

(a) $\left[-r, \frac{-2}{3}r\right]$ (b) $\left[\frac{2}{3}r, r\right]$

:c) $\left[-\frac{r}{3}, \frac{r}{3}\right]$ (d) $\left[\frac{1}{3}r, r\right]$

iven a set A that is strictly partially ordered, in vhich every simple ordered subset has an upper ound, A itself has a maximal element ———— his result is known as.

a) Urysohn lemma

b) Zorn's lemma

:) Tube lemma

l) The sequence lemma

. collection B of subsets of X is said to be ountable locally finite it B can be written as

) the countable union of collections B_n each of which is locally finite

) the countable union of open sets

) the countable union of no where dense subsets

) the countable union of collections B_n each of which is locally connected

9. The interior of Q as a subset of $\mathbb{R}$ is

(a) ϕ (b) Q

(c) $R-Q$ (d) $\mathbb{R}$

10. The interior of $Q \times R$ as a subset of $\mathbb{R}_2$

(a) $R \times R$ (b) $Q \times R$

(c) $Q \times Q$ (d) ϕ

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable sub collection covering X.

Or

(b) Prove that a subspace of a regular space is regular.

12. (a) Show that every compact Hausdorff space is normal.

Or

(b) Define a completely regular space. And Show that a normal space is completely regular and a completely regular space is regular.

13. (a) Let X be a compact Hausdorff space. Show that X is metrizable if and only if X has a countable basis.

Or

(b) Show that the Tietze extension theorem implies the Urysohn lemma.

14. (a) Let A be a locally finite collection of subsets of X. Prove that the collection B of the closures of the elements of A is locally finite.

Or

(b) Show that if X has a countable basis, a collection A of subsets of X is countably locally finite if and only if it is countable.

15. (a) Define a Baire space. Give an example of a topological space which is not a Baire space (with justification).

Or

(b) Prove that any open. Subspace Y of a Baire space X is itself a Baire space.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that a subspace of a second countable space is second countable but a subspace of Lindelot space need not be Lindelot.

Or

(b) Let X be a topological space. Let one-point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subseteq U$.

17. (a) Prove that every regular space with a countable basis is normal.

Or

(b) State and prove the Urysohn lemma.

18. (a) Prove that every regular space X with a countable basis is metrizable.

Or

(b) State and prove Tietze extension theorem.

19. (a) State and prove Tychonoff theorem.

Or

(b) Let X be a metrizable space. If A is an open covering of X, prove that there is an open covering ε of X refining of that is countably locally finite.

20. (a) If X is a compact Hausdorff space of a complete metric space, prove that X is a Baire space.

Or

(b) Let X be a space; let (Y, d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$ where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.
